

Wave Equation Based Imaging Techniques

Sudhakar Yernneni , Dheeraj Bhardwaj
Centre for Development of Advanced Computing
Pune University Campus, Pune - 411 007 (India)

&

Suhas Phadke
Western Atlas , Houston (USA)

Abstract

In This article we have discussed about the wave equation based imaging techniques using parallel computing. The mathematical formulation of extrapolation equation for ω -x migration is given. Parallel Implementation of 3D ω -x migration is discussed in detail. Impulse response is shown as an example.

Introduction

Seismic imaging techniques occupies significant role in the structural delineation. It gives high resolution picture of earths subsurface. Current advances in 2D and 3D data acquisition have increased the input data volume by several folds. Processing methods have also changed for high resolution processing which amounts to an increase in the computational efforts that is beyond the scope of currently available serial architecture machines. As there is an explosion in the processing power of computers, there is an explosion in the requirements as well. All over the world it has been realized that parallel processing is the only answer to this challenge and it is fortunate that seismic data processing is an ideal application for parallel architecture machines.

Wave equation based methods (Phadke et. al., 1998) are widely recognized in the industry as more accurate while providing finer detailed geological features than other conventional methods. However, the wave equation based methods are computationally more expensive but suitable for currently available parallel computers.

Wave Equation Based Imaging

Seismic data acquisition involves recording the wavefield at the Earth's surface by the sensors placed along a line or in an aerial pattern. This recorded wavefield is

used as an initial condition or boundary condition for seismic migration. The extrapolation of the recorded wavefield is governed by the wave equation. The two most important steps in migration are *extrapolation* and *imaging*. Extrapolation involves numerical reconstruction of the wavefield at depth from the wavefield recorded at the Earth's surface. Imaging is the process that allows one to obtain the local reflection strength from the extrapolated data in depth and create an image of the subsurface reflectors.

Poststack migration methods are applied to zero-offset data and are based upon the exploding reflector concept. Migration can be performed in time or in depth. In the presence of strong lateral velocity variations, time migration followed by time to depth conversion does not image the reflected energy to its true subsurface position. Depth migration is essential in these cases. Depth migration compensates for ray bending, lateral velocity pull-ups and structure. A natural advantage of depth migration is that the output image is displayed in depth and therefore can directly be utilized for geologic interpretation

One-way wave equation for downward extrapolation of the wavefield in 3D is derived from the 3D acoustic wave equation

$$\frac{\partial P^2}{\partial x^2} + \frac{\partial P^2}{\partial y^2} + \frac{\partial P^2}{\partial z^2} = \frac{1}{v^2} \frac{\partial P^2}{\partial t^2} \quad (1)$$

A Fourier transformation with respect to x , y , z and t gives us the 3D dispersion relation

$$k_z = \pm \frac{\omega}{v} \sqrt{1 - \frac{v^2 k_x^2}{\omega^2} - \frac{v^2 k_y^2}{\omega^2}} \quad (2)$$

By keeping only the negative square root term and taking an inverse Fourier transform with respect to z , we obtain one-way wave equation in 3D

$$\frac{\partial P}{\partial z} = -i \frac{\omega}{v} \left(\sqrt{1 - \frac{v^2 k_x^2}{\omega^2} - \frac{v^2 k_y^2}{\omega^2}} \right) P \quad (3)$$

One way wave equation in ω - x domain is derived by first approximating the square root and then taking an inverse Fourier transform with respect to x and y . Using

$$X = \frac{k_x v}{\omega} \quad \text{and} \quad Y = \frac{k_y v}{\omega} \quad (4)$$

the square root term in (2) can be approximate by (Brown, 1983)

$$R = \sqrt{1 - X^2} + \sqrt{1 - Y^2} - 1 \quad (5)$$

Following (Bunks, 1992), we can write the equation (5) as

$$R = \rho - \frac{\beta X^2}{1 - \alpha X^2} - \frac{\beta Y^2}{1 - \alpha Y^2} \quad (6)$$

where ρ , α and β will be determined by solving an optimisation problem.

A 45 degree approximation to (6) given by

$$R = 1 - \frac{0.5X^2}{1 - 0.25X^2} - \frac{0.5Y^2}{1 - 0.25Y^2} \quad (7)$$

Now substituting (7) in (2) and using the definition of retarded wavefield (Claerbout, 1985) , we obtain the one-way depth extrapolation equation in 3D

$$\frac{\partial Q}{\partial z} = -i\omega \left\{ \frac{\rho}{v(x,z)} - \frac{1}{v(x,z)} \right\} Q + \left(i \frac{\beta k_x^2 \frac{v}{\omega}}{1 - \alpha k_x^2 \frac{v^2}{\omega^2}} \right) Q + \left(i \frac{\beta k_y^2 \frac{v}{\omega}}{1 - \alpha k_y^2 \frac{v^2}{\omega^2}} \right) Q \quad (8)$$

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Thin Lens Term

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Diffraction Terms

Thin lens term has a straightforward analytic solution, whereas diffraction terms are solved by finite difference method using a splitting technique. The splitting method for solving the diffraction term is also called onepass migration method. This method works very well for handling strong lateral velocity variations.

For the one pass method the field is downward continued alternately along the x and y directions for each depth step. The differential equations in ω -x domain for downward continuation in x and y directions are given by

$$\left(1 + \frac{\alpha v^2}{\omega^2} \frac{\partial^2}{\partial x^2} \right) \frac{\partial Q}{\partial z} - i \frac{\beta v}{\omega} \frac{\partial^2 Q}{\partial x^2} = 0 \quad (9)$$

$$\left(1 + \frac{\alpha v^2}{\omega^2} \frac{\partial^2}{\partial y^2} \right) \frac{\partial Q}{\partial z} - i \frac{\beta v}{\omega} \frac{\partial^2 Q}{\partial y^2} = 0 \quad (10)$$

The imaging part is again the summation of all the frequencies at $t=0$ at each depth step.

$$P(x,y,z,t=0) = \sum_{\omega} P(x,y,z,\omega) \quad (11)$$

Parallel Implementation of Migration Algorithm

The depth migration algorithm in ω - x domain is inherently parallel in terms of frequencies. The parabolic approximation of the wave equation in frequency-space domain has decomposed the wave field into monochromatic plane waves that are propagating downwards. Therefore, each frequency harmonic can be extrapolated in depth independently on each processor and there is no need of intertask communication. One can introduce parallel task allocation into each frequency harmonic component with the ultimate goal being to have as many processors as frequencies. At each depth step all frequency components after extrapolation are summed up (Imaging Condition) to give the migrated image. Computations, communications and I/O are overlapped in order to achieve the efficiency and speed.

The migration codes are analogous to a client-server system, where there is one client with multiple servers. One can also think of it as Master-Worker system where Master works as the manager and assigns tasks to his Workers. The job of the Master is to provide the required parameters and data to all the workers and distribute workload properly, so that idle time of the workers is minimum. Also at the end Master should collect the finished work from all the workers, compile it and store it in a proper manner. One of the processors acts as Master and the Worker tasks are assigned to different processors.

A flow chart of the parallel implementation is shown in Figure 1. The figure only shows one Master task and one Worker task, but in reality there are many Worker tasks. All the Worker tasks communicate with Master task in an identical fashion as shown in the figure. For communication between Master and Worker, i.e. in order to exchange data between Master and Workers over the network, we make use of message passing model called PVM (Parallel Virtual Machine) / MPI (Message Passing Interface). The PVM/MPI systems are the software frameworks for concurrent computing in a networked environment. The PVM (Geist et. al., 1994)/MPI(Pacheco, 1997) models are the set of message passing routines which allow data to be exchanged between tasks by sending and receiving messages. The messages are coded with origin, destination and identification tags in order to avoid any mix up during network transmission.

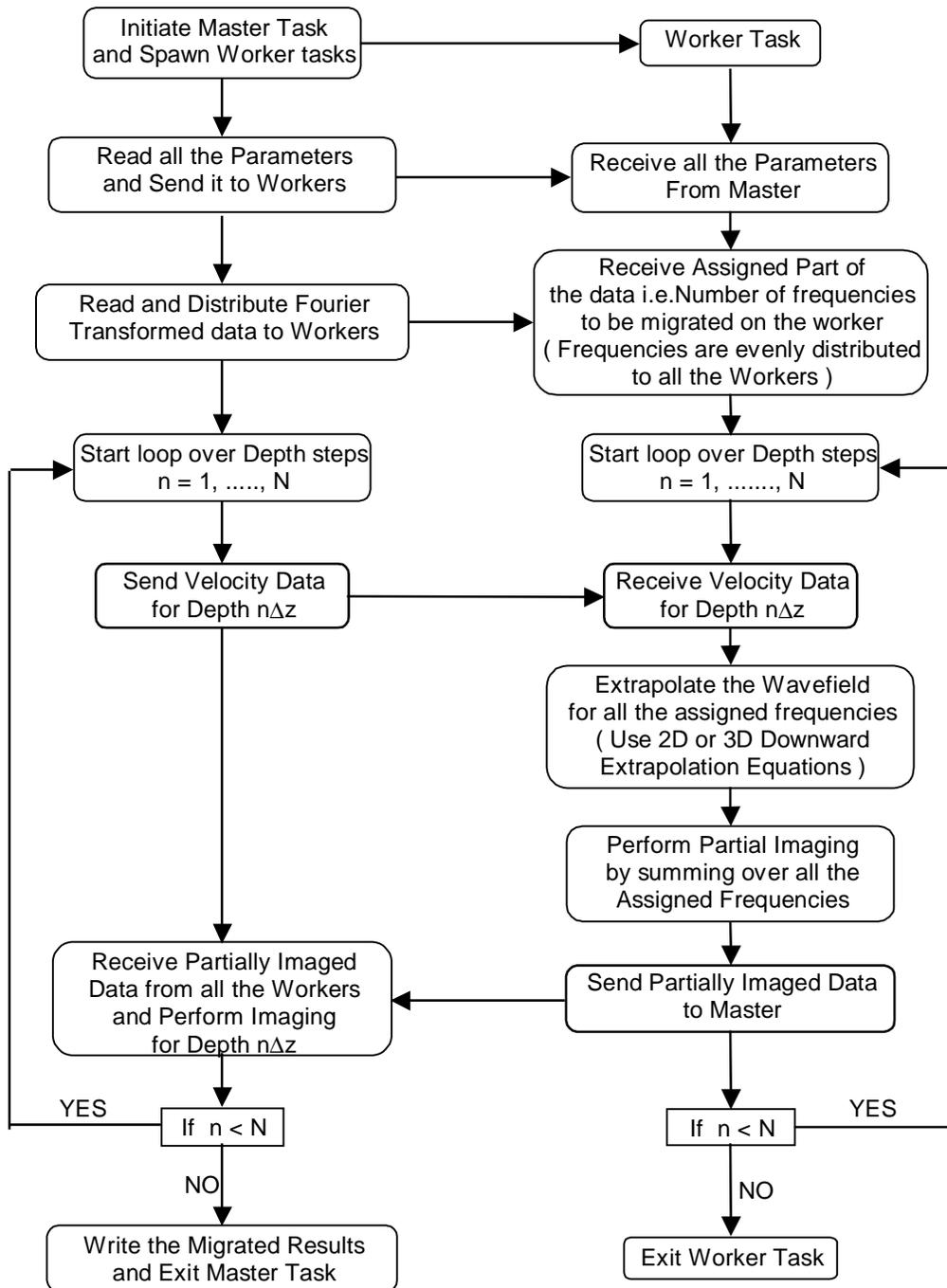


Figure 1 : Flowchart of the ω -x migration algorithm

Impulse Response

The horizontal slices of the 3D impulse response of the migration operator for 45 degree approximation are shown in Figure 2.

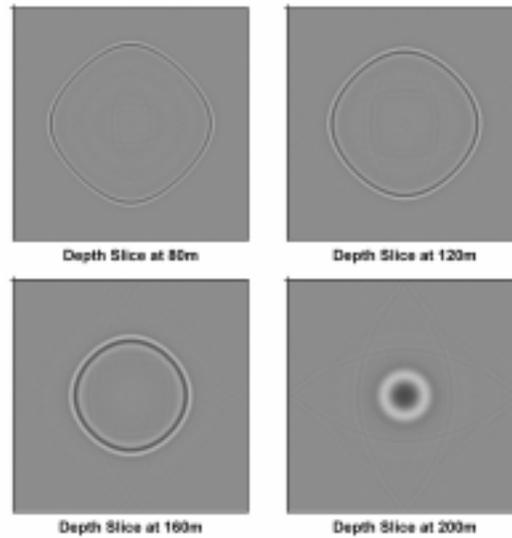


Figure 2 : Horizontal slices at different depths for the 3D impulse response of the migration operator.

In order to calculate the impulse response of the 3D migration operator shown in Figure 2, a bandlimited Ricker wavelet with a dominant frequency of 30 Hz was used. The other parameters are $\Delta x = 5.0\text{m}$, $\Delta y = 5.0\text{m}$, $\Delta z = 2.0\text{m}$, $\Delta t = 0.002\text{ sec}$, $N_x = 251$, and $N_z = 120$.

Conclusion

The discussed ω - x depth migration method images the earth's subsurface very well even when there are lateral velocity variations. So, in the case of continental margin boundary delineation, where there may exist sharp lateral velocity variations, this method can be used for high resolution imaging. Parallel implementations have been carried out on PARAM 10000 using PVM (Parallel Virtual Machine) & MPI (Message Passing Interface) message passing libraries. Use of PVM / MPI enable the portability of the software across various parallel machines.

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